Cognitive Complexity: Algebra 1

Using the framework below (adapted from Depth-of-Knowledge for Four Content Areas, Norman L. Webb, March 28, 2002), determine the Cognitive Complexity for your students of each task and *give a brief reason for your choice*.

**Level 1 (Recall)** includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Also identify the Standards for Mathematical Practices (SMP) students may use while engaging in the task.

**Level 2 (Skill/Concept)** includes the engagement of some mental processing beyond a habitual response. A Level 2 task requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps.

**Level 3 (Strategic Thinking)** requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning.

**Level 4 (Extended Thinking)** requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.

|  |  |  |  |
| --- | --- | --- | --- |
| **Task** | **Cognitive Complexity** | **Reason** | **SMP** |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |
| E |  |  |  |
| F |  |  |  |
| G |  |  |  |
| H |  |  |  |
| I |  |  |  |
| J |  |  |  |
| K |  |  |  |
| L |  |  |  |
| M |  |  |  |
| N |  |  |  |
| O |  |  |  |
| P |  |  |  |

Task A

Compare the expressions using <, >, or =.

|  |  |
| --- | --- |
| 1. or 2. or 3. or 4. or 5. or 6. or 7. or 8. or | 1. or 2. or 3. or 4. or 5. or 6. or 7. or |



Task B

Write equivalent expressions for each expression below, using bases and exponents.

Task C

Write an expression, written as a sum, that is equivalent to .

Write another expression equivalent to but the expression is written as a quotient.

Write another expression equivalent to but the expression should contain a negative exponent.

Write another expression equivalent to that you think will be different from anyone else’s expressions.

Task D

At the beginning of January, Susita had some money in her checking account. At the end of each month she deposits enough to double the amount currently in the account. However, she has a loan to pay off, requiring her to withdraw $10 from the account monthly (immediately after her deposit).

1. Assuming January is the first month, write an equation that describes the amount of money in Susita’s account at the end of the *n* th  month, *S*(*n*) , in terms of the amount of money in Susita’s account at the end of the (*n*−1) th  month, *S*(*n*−1) .
2. At the end of May, Susita had $2 left in the account. How much did she have at the beginning of January?

(Source: Illustrative Math Project)

Task E

Given the number line below, place the following numbers as accurately as possible. If you cannot place them accurately, create another number line that would show where each number is in relation to 0 and 102.

101, 10-1,103, 



Task F

Store the value -2 in A and 3 in B in your calculator. Predict the results of the expressions listed below, then find the calculator result.

|  |  |  |
| --- | --- | --- |
| **Expression** | **Predicted result** | **Calculator result** |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
| -B2 |  |  |
| (-B)2 |  |  |
|  |  |  |

Jane typed  into her calculator and the calculator gave a result of 0.125. What number was stored in A?

Task G

Miguel and his sister Anna each deposited the same amount of money into investments. Miguel’s investment earned 5% per year, eventually doubling his original investment, at which time he withdrew all his money (principal and interest) from the investment. Unfortunately, Anna’s investment lost 5% per year and she withdrew what was left when her original investment was cut in half.

1. Without calculating, who withdrew their money sooner? Explain your reasoning.
2. Solve the problem, and show and explain your reasoning. Is the answer different from what you expected? If so, reconsider your reasoning above.

Task H

Find values of *x* that make the statements true, and justify your answers.

1. Give two values of *x* so that < is a true statement.
2. Give two values of *x* so that > is a true statement.
3. Give two values of *x* so that < is a true statement.
4. Give two values of *x* so that > is a true statement.
5. Give two values of *x* so that  is a true statement.
6. Find values for *x* and *y* so that  is a true statement.

Task I

Simplify without a calculator.

1. 

2. 

3. 

4. 

Compare  and  and determine how many times greater one expression is than the other.

Task J

1. Simplify by writing as an equivalent expression. State which rule you use in each step.

Task K

Use the meaning of rational exponents to simplify. Explain how you thought about it.

1. What does  mean?
2. Approximate  without a calculator and explain your reasoning.
3. Use the meaning of  to find the product . Explain how you used the meaning.
4. Use rules of exponents to simplify the following:
   1. 
   2. 
   3. 
   4. 

Task L

The graphs of  and  are shown.

 

Use the graphs to compare the expressions using <, >, or =.

1.  or 

2. or 

3.  or 

4.  or 

5.  or 

6.  or 

7.  or 

Task M

1. Look for a pattern and sketch the square at Step 4.

|  |  |
| --- | --- |
| Step 0:    Step 1:    Step 2:    Step 3:    Step 4 | 1. What new portion of the big square is shaded at Step 10? Explain how you know. 2. What new portion of the big square gets shaded in the nth step? 3. Suppose this process continues infinitely many times. Make a conjecture about what portion of the big square is eventually shaded and why you think that. |

Task N

On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

1. When will the lake be covered half-way?
2. On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?
3. On June 29, a clean-up crew arrives at the lake and removes almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

d. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake if the cleanup crew does not come on June 29.

(Source: Illustrative Mathematics Project)

Task O

Mark works in a lab and has been given the task of measuring the growth of a certain bacteria. He measures the number of bacteria in the colony at noon and then each hour afterwards. Unfortunately, he found a stray cat on the way to work and decided to let the cat hang out in the lab all day. When Mark was away from his desk, the cat walked across his computer keyboard and erased the data he collected every hour from noon until 6:00. Mark goes ahead and measures the bacteria colony at 7:00 and 8:00 before he heads home for the day. At 7:00 he measures 17.68 thousand bacteria and at 8:00 he measures 25.64 thousand bacteria. Help Mark find an exponential function that models the growth of the bacteria colony, and justify your model.

How many bacteria were in the colony at noon?

Task P

Jerome was given a gift of cash by his grandparents on his 15th birthday. He decided to deposit the money into a bank and let it earn interest until he graduates from high school. He researched the interest rates and compounding periods at three banks and came up with three options. The expressions below show the amount of money that will be in the accounts after *t* years if he initially deposits *P* dollars.

Bank A: 

Bank B: 

Bank C: 

1. Compare the annual interest rates at each bank. Explain how you know.
2. Explain the meanings of the expressions  and  in terms of the context.
3. His friend, Joni, says that he should just choose the bank with the highest annual interest rate. Is Joni right? Explain.