Students were given the following formative assessment task. The purpose of the assessment was to see if students could see structure in expressions that were quadratic, linear, or exponential, and had an idea of the general shapes of the graphs of these functions.

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| Match pairs of functions that have the most similar graphs. Briefly explain how you matched them.a.  b.  c. d.  e.  f. g.  |

Three students’ answers follow, with

1) an evidence-based description of the answer,

2) analysis of students’ mathematical ideas evidenced in the answer,

3) ideas of how the response could be used to further student thinking.

**Student A:**

$f\left(x\right)=3+4x$ and $f\left(x\right)=3+4^{x}$ have similar graphs, and $f\left(x\right)=2x$ and $f\left(x\right)=2^{x}$ also have similar graphs. (There was no further explanation.)

1. Evidence-based description of the answer: The student notices that the 3 and the 4 are in the same places in the first two functions, and that the x is in the same place just after the 4, but does not seem to notice, or perhaps realize, or know that it makes a difference, that the x is a multiplier in one function, but an exponent in the other. The pairing of the last two functions is consistent with this view.
2. Analysis of students’ mathematical ideas evidenced in the answer: The work does not demonstrate knowledge of the difference between graphs of exponential functions and linear functions. The student may not understand that a number raised to an exponent is different from a number multiplied by the same number.
3. Ideas of how the response could be used to further student thinking: This student could be asked to graph and examine the tables of the first two functions matched, then asked to explain how and why they are different. Finally, they could be asked to make a conjecture about the next pair of functions they matched, explaining why, and check this conjecture on their graphing calculator. This student also needs more experience noticing the differences in graphs of the same type of function when only numbers in the functions are changed. For example, only graphing exponential functions with changed numbers.

**Student B**:

$f\left(x\right)=3+4x$ and $f\left(x\right)=3+4x^{2}$ have similar graphs, “because both functions have a 3 and a 4.”

1. Evidence-based description of the answer: This student, like Student A, focuses on similarities between numbers, rather than structure (the operations, especially squaring). However, while he or she indicates that it is because both functions have a 3 and a 4, the student does not include  which also only includes the numbers 3 and 4.
2. Analysis of students’ mathematical ideas evidenced in the answer: The work indicates that the student does not distinguish between x and x2, but does distinguish between each of these and a function with an exponent of x.
3. Ideas of how the response could be used to further student thinking: This student could be asked to create some functions with the same numbers but with exponents of 1 (not showing) and 2, on the x, and asked to compare graphs and tables (with a graphing calculator). He or she should be asked to explain how the graphs are different and what causes the difference. In particular, asked to examine why the output values of the two functions are changing at different rates, by comparing the tables.

**Student C:**

$f(x) = 2x$ and $f\left(x\right)= 2x^{-1}$ have similar graphs, “because the exponent of -1 on the x in the second function has no effect.” (The student also showed several correct pairings of functions.)

1. Evidence-based description of the answer: This student described that an exponent of -1 on a variable has no effect. S/he does not appear to have meaning for the -1 exponent as the multiplicative inverse of x, or as 1/x.
2. Analysis of students’ mathematical ideas evidenced in the answer: The student does not have correct meaning for an exponent of -1. s/he could believe that a -1 exponent is similar to an exponent of 1, in that an exponent of 1 “does nothing” (rather than as implied).
3. Ideas of how the response could be used to further student thinking: Like the other students, this student could be asked to use a graphing calculator to examine the difference between functions whose only difference is the exponent of -1, and asked to explain why (although graphing reciprocal functions may be beyond algebra 1). Further exploration, explanations, and questioning could focus on the meaning of -1 as an exponent.