**Goals:** Solving problems in multiple ways to make connections is a way to teach for coherence. Identify how the math in the task addresses the SMP. Teachers should strive to *notice* the SMP they used and that used by their peers. How would they respond to help their students become more aware of the SMP they were using and the power of using them?

**Introduction:** Lots of math is counting, counting singles, counting how many times you added, how many times you multiply, etc. And so in math we create…structures for these counting principles. The questions is, **can we see the structures** for expediting our counting practice?

**Counting Exercise Instructions**

Slide 1: Triangle Image:

What are the different shapes you see in the figure?

Slide 2:

Problem: Without actually counting every triangle, find an efficient way to count the triangles in the figure:



Quiet Individual work time: 5 minutes or until every person has at least 1 method running.

Groups discuss methods:

1. How many different ways of seeing patters people found
2. Dead Ends that people found

Slide #3

Make a poster with your **counting method**, **number expression** that led to answer (5 minutes to make) and your **algebraic rule**

**Place posters around room**.

**Variable Exercise Instructions**

Each person takes the “Variable Scaffold”

**Slide #4**

* As a group, select a counting method/number expression that you explored.
* Test the counting method to see if it would work on a different size pyramid such as the one here.
* Can you use the counting method to **write a rule** for finding the number of triangles in any size pyramid?



**Possible Reflection Questions**

SMP related

In your groups discuss…

1. What mathematical practices did you use?
2. What were the behaviors that your were doing that exemplified those practices?
3. What specific behaviors was the teacher doing that elicited those behaviors?
4. What about the task elicited those behaviors?

Task creation related

1. Over the summer we did a very similar problem called staircase. How was your response to this problem different than the one we did this summer?
2. What specific behaviors was the teacher doing that elicited those behaviors?
3. What about the task elicited those behaviors?



Ways students might solve it:

1. Each row adds the next odd number of triangles, with figure 2 adding 3 and figure 3 adding 5, so the sum is 1+3+5+…+(2n-1) where n is the Figure number. The last term will not be easy for Algebra 1 students to figure out, but they may be able to verbalize it, and could be encouraged to be precise. If they can describe the bottom row in terms of the diagram by adding the pointing up triangles, n, to the pointing down triangles, n-1, this will give them 2n-1. There will be other ways to see it, too.



2. Each new figure can be shaded so that all the ‘pointing up triangles’ are shaded the same color, and all the pointing down triangles are not shaded. So the sum of the pointing down triangles and the pointing up triangles is shown in the table:



(Note that the numbers of Pointing Down and Pointing up triangles are triangular numbers, which suggests that square numbers are the sum of two consecutive triangular numbers. This is a pretty cool connection to the Staircase problem since those were the triangular numbers.)

|  |  |  |
| --- | --- | --- |
| Figure # | Pointing down + Pointing up | Total |
| 1 | 0+1 | 1 |
| 2 | 1+3 | 4 |
| 3 | 3+6 | 9 |
| 4 | 6+10 | 16 |
| 5 | 10+15 | 25 |
| n | (n-1)st +nth triangular numbers | Simplifies to n2 |

3. Recursive: From (n-1)st stage to nth stage, we add the odd number 2n-1 (can see it as n pointing up and n-1 pointing down, so 2n-1). So, we get the number in the previous term plus the number of triangles in the bottom row  which also simplifies to n2. Note that we used a closed form for the previous term.

4. Put two nth stages together to form a parallelogram (how do we know it is a parallelogram?) Now it is a bit easier for some to see the number of triangles, since in Figure 3, there are 6 small triangles in each of 3 rows. We can imagine this as a 3 by 3 array of rhombi, each cut in half. Thus, there are 2n2 triangles, but we doubled the figure, so there are only n2 triangles in the nth figure.

 